**Lab II Fourier Analysis**

Fall 2019

**Objectives**

1. Review of continuous-time Fourier series and Fourier transform
2. Windowing effects on Fourier transform
3. Discrete-time Fourier transform
4. Discrete Fourier transform and its fast algorithm
5. Application of Fourier analysis
6. **Review of Fourier Series and Fourier Transform**

CTFS and CTFT can only be approximately calculated in the numerical way. For example, you can sample a limit time duration of the CT signals with a high enough sampling rate. Strictly to say, if the CT signals are not band-limited and energy-limited, we cannot exactly represent them with samples. Then, we can rewrite CTFS and CTFT formula into their numerical forms and calculate Fourier coefficients. Are these numerical results equal to the corresponding theoretical ones? why?

**Ex. 1.1**

1. Create a sub-function **x = mySin (*f*, *t*, *A*, *phi*)** which produces samples in terms of Eq. (1).



Represent *x*(*t*) with *f*0 = 5, *f***s** = 1000, *A* = 1.5, = π/6, and plot the time-domain signal with five periods.

1. Calculate Fourier series of *x*(*t*)with *k* = -10:10, represent the FS result *X*(*k*)in an appropriate way and try to illustrate how the result can be considered correct according to the frequency spectrum.
2. Plot and compare the numerical results with the theoretical ones.
3. Apply the Parseval’s formula to above *x*(*t*) and *X*(*k*) to calculate the power, tell whether the results show the time-frequency energy equivalence; If there is an error, try to explain the cause of the error and find out the possible solution to minimize the error.

**Ex. 1.2**

1. Create the sub-function [**x, t] = mySquare (*D*, *B* *H*, *fs*, *N*)**, where *D* stands for the cycle, *B* for the duty cycle, *H* for the amplitude, *fs* for the sampling frequency, *N* for the number of cycles. This square function should be even function. Represent the CT *x*(*t*) with *D* = 6, *B*= 0.5, *H* = 1, *fs* = 500 Hz and *N* = 3.
2. Create the sub-function **x= myCTFT (*x*, *t*, *f*)**, where *x* stands for the time-domain signal, *t* for the time vector and *f* for frequency vector. Calculate the FT of *x*(*t*) with *f*= -10~10and show the modules of  *.*
3. Deduce the theoretical results of the Fourier transform of *x*(*t*).
4. Plot and compare the numerical FT results with the theoretical ones.
5. Apply the Parseval’s formula to above*x*(*t*) and  to calculate the energy, tell whether the results show the time-frequency energy equivalence; If there is an error, try to explain the cause of the error and find out the possible solution to minimize the error.
6. **Windowing effects on FT**

**Ex 2.1**

1. Create the continuous-time harmonics *x*(*t*) with *f*s = 500 Hz, *f*1 = 5, *A*1 = 1, *φ*1= *0*,, , *A*2 = 0.1, *φ*2 = 0, D = 0.5 and 0 ≤ *t* ≤ D, according to the Eq. (3).



1. Calculate the FT results of *x*(*t*). Demonstrate the amplitudes *A*1 and *A*2 and harmonic frequencies *f*1 and *f*2 from the spectrum. Are the values equal to the ones of the original parameters?
2. Truncate the signals with D=1 and 5, respectively, show the two windowed signals *x*1(*t*) and *x*2(*t*). Calculate and plot the FT of the signals. Indicate the amplitudes *A*1 and *A*2 and harmonic frequencies *f*1 and *f*2 and demonstrate the windowing effect on the spectra.
3. Apply the rectangle window and Hanning window with width D = 5to truncate the signals and show the two windowed signals *x*w(*t*). Calculate and plot the modulus of FT of *x*w(*t*). Indicate the amplitudes *A*1 and *A*2 and harmonic frequencies *f*1 and *f*2.
4. Compared to the FS results of sinusoidal signals, demonstrate the smearing and leakage effects on identification of the spectrum peaks.
5. **Discrete-time Fourier Transform**

**Ex. 3.1**

1. Sample the continuous-time signal x(t) in Ex. 1.2 with the time intervals *T* = 0.1, 0.5, 1, respectively. Plot *x*(*nT*) with respect to *t* with different sampling intervals.
2. Calculate the corresponding Fourier transform of *x*(*nT*) and plot their modulus curves in the frequency region from -10 to 10. Indicate the Nyquist interval of these spectra.
3. Compare the theoretical results with the numerical ones. Demonstrate the difference between CTFT and DTFT.
4. **Discrete Fourier Transform**

**EX4.1**

1. Create a triangular wave *x*[*n*], see Eq. (1), compare DTFT and DFT of *x*[*n*], and then indicate the computational resolution of DFT.



1. Define a cosine signal *y*[*n*], as shown in Eq. (2), where *f*0=1/16, *A* = 1.5, and define a rectangle window *w*[*n*], as shown in Eq. (3). The truncated cosine signal *yw*[*n*]=*y*[*n*]·*w*[*n*] has a length of *N*+1, where *N* = 16. Calculate and show DFT and fft (MATLAB function) in digital frequency .





In order to investigate the computational time of DFT and fft with respect to the window length, use a tic-toc function to evaluate the computational time with *N* = [10, 50,100, 500, 1000, 5000]. Show the curve of computational time with respect to *N*.

In order to investigate the relationship between fft algorithm and the power of 2 points, use a tic-toc function to evaluate the computational time with *N* = [1000, 1023, 4000,4095]. Show the curve of computational time with respect to *N*. Tell which spends less time.

**5 Application**

The ideal electricity wave is a harmonic signal with the frequency of 50Hz and the amplitude of 220V, denoted as E0 = 220@50. Due to the interference of some high-power inverters, the practical electrical waveform includes the fundamental frequency *e*1 = 218@50, the high frequency harmonics *e*2 = 2@100,*e*3 = 0.5@150 and others, and inter-harmonics e4 = 5@55.



To enable these frequency components could be found in the frequency spectrum, choose proper window and sampling frequency to construct a signal *x*(*t*), as shown in Eq. (4). Then calculate the fft of *x*(*t*) and mark the peak of *e*n in spectrum. Notice that the frequency error should be less than 1% and the amplitude error should be less than 5%.

Try to use DTFT to calculate these frequency components of x(t) and determine whether the result meets the error limitation.